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## *EUROPEAN FEL Design Study*



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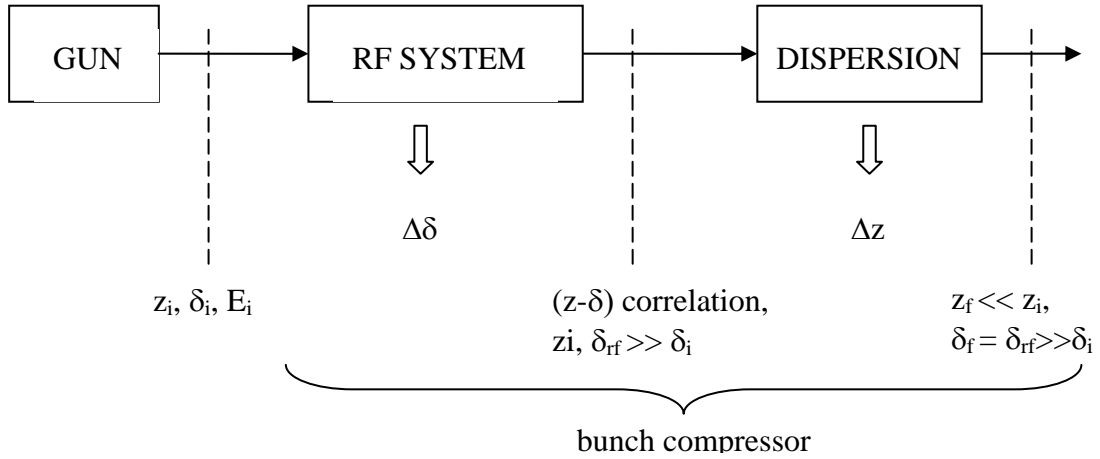
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### Definition of variables and parameters

We define a magnetic bunch compressor as a system composed of an rf accelerator, which may include several distinct accelerating sections, followed by a dispersive region, i.e. a magnetic chicane (see, Figure 1). The rf system accelerates the electron beam off-crest and generates a correlation in the longitudinal phase space that is an energy chirp ( $\Delta\delta$ ). The linear momentum compaction – the  $R_{56}$  term in the transport matrix of the chicane – relates the particle energy to its path length, so that higher (lower) energy particles cover shorter (longer) paths and the bunch length is finally reduced ( $\Delta z$ ). As a result, the compressed beam at the chicane exit has a large correlated energy spread and a reduced length, thus a higher peak current.



**Figure 1.** Schematic of a magnetic bunch compressor composed of a booster linac and a dispersive region, which couples longitudinal and transverse dynamics through dispersion in the bending plane.

If  $\delta$  is the energy deviation of a particle from the central momentum  $p_0$ , it may be expanded in a Taylor series as function of the longitudinal position  $z$  along the bunch:

$$\delta = \frac{\Delta p}{p} \cong \delta_i + \left[ \frac{\partial \delta(p_0)}{\partial z} \right]_{z=z_0} \cdot z + \frac{1}{2} \left[ \frac{\partial^2 \delta(p_0)}{\partial z^2} \right]_{z=z_0} \cdot z^2 + O(z^3) \quad [1]$$

or in symbols:

$$\delta = \delta_i + \alpha_{rf}(p_0) \cdot z + \frac{1}{2} \beta_{rf}(p_0) \cdot z^2 + O(z^3) \quad [2]$$

If the rf system provides an energy gain  $\Delta U_{rf} = U_{rf} \cos(\varphi + \frac{s}{\lambda_{rf}})$ , then the parameters

$\alpha_{rf}$  and  $\beta_{rf}$  in [2] are defined by<sup>1</sup>:

$$\alpha_{rf}(p_f) = -\frac{U_{rf} \sin \varphi}{E_f} \cdot \frac{1}{\lambda_{rf}} \quad [3]$$

$$\beta_{rf}(p_f) = -\frac{U_{rf} \cos \varphi}{E_f} \cdot \frac{1}{\tilde{\lambda}_{rf}^2} \quad [4]$$

$\alpha_{rf}$  is the linear energy chirp,  $\beta_{rf}$  is the nonlinear one. For small rf phases,  $\alpha_{rf}$  ( $\sim \phi$ ) mainly represents the total energy chirp of the beam, while  $\beta_{rf}$  ( $\sim \phi^2$ ) is the rf curvature.

A similar treatment can be applied to the path length variation in the chicane that is the variation of the longitudinal position of a particle inside the bunch:

$$\Delta z \cong \delta_i + \left[ \frac{\partial z}{\partial \delta(p_0)} \right]_{\delta=0} \cdot \delta(p_0) + \frac{1}{2} \left[ \frac{\partial^2 z}{\partial \delta(p_0)^2} \right]_{z=0} \cdot \delta(p_0)^2 + O(\delta^3) \quad [5]$$

or in symbols:

$$\Delta z = R_{56}(p_0) \cdot \delta + T_{566}(p_0) \cdot \delta^2 + O(\delta^3) \quad [6]$$

The path length  $s(\theta, \delta)$  is a function of the bending angle and of the particle energy deviation; in a symmetric chicane it is proportional to

$\theta^2(\delta) = \frac{\theta_{\delta=0}^2}{(1+\delta^2)} \cong \theta^2(1-2\delta+3\delta^2+\dots)$  and comparing with [6] it may be evaluated:

$$R_{56} = \left[ \frac{ds(\theta, \delta)}{d\delta} \right]_{\delta=0} \propto -2 \quad \text{and} \quad T_{566} = \left[ \frac{d^2s(\theta, \delta)}{d\delta^2} \right]_{\delta=0} \propto 3 \quad [7]$$

Thus, the ration between 1<sup>st</sup> and 2<sup>nd</sup> order momentum compaction in a pure magnetic chicane of rectangular dipoles is fixed to:

$$\frac{T_{566}}{R_{56}} = -\frac{3}{2} \quad [8]$$

A more general definition of the momentum compaction is given in the optics formalism. If  $x$  is the transverse coordinate in the bending plane and  $\eta_x$  the horizontal dispersion, the particle motion can be approximated as a linear superposition of betatron and dispersive motion:

$$x \cong x_\beta + x_\varepsilon = x_\beta + \eta_x \delta + \eta_x'' \delta^2 \quad [9]$$

and it results to be:  $R_{56} = -\int \frac{\eta_x(s)}{\rho(s)} ds$  and  $T_{566} = \int \frac{\eta_x''(s)}{\rho(s)} ds$  [10]

## Beam transport

Beam transport through the layout of Figure 1 can be represented by matrix formalism. The matrix for the longitudinal beam transport in the rf system is:

$$[rf] = \begin{pmatrix} 1 & 0 \\ \alpha_{rf} + \frac{1}{2}\beta_{rf}z_i & 1 \end{pmatrix} \quad [11]$$

The matrix for the dispersive region is:

$$[disp] = \begin{pmatrix} 1 & R_{56} + T_{566}\delta_{rf} \\ 0 & 1 \end{pmatrix} \quad [12]$$

Thus, the final particle position along the bunch and its energy deviation are obtained through the usual matrix multiplication:

$$\begin{pmatrix} z_f \\ \delta_f \end{pmatrix} = [ch][rf] \begin{pmatrix} z_i \\ \delta_i \end{pmatrix} \quad [13]$$

We finally obtain eqs. [14] and [15] for the final longitudinal position and energy deviation, respectively:

$$z_f = \underbrace{R_{56}\delta_i + T_{566}\delta_i^2}_{\text{dispersive optics to 2}^{\text{nd}} \text{ order}} + \underbrace{(1 + \alpha_{rf}R_{56})z_i}_{\text{coupling term to 1}^{\text{st}} \text{ order}} + \underbrace{(\alpha_{rf}^2 T_{566} + \frac{1}{2}R_{56}\beta_{rf})z_i^2 + 2\alpha_{rf}T_{566}z_i\delta_i}_{\text{coupling terms to 2}^{\text{nd}} \text{ order}} \quad [14]$$

$$\delta_f = \delta_i + \alpha_{rf}z_i + \frac{1}{2}\beta_{rf}z_i^2 \quad [15]$$

Since  $T_{566}$  applies to  $\delta_{rf}$  in [13] and  $\delta_{rf}$  is determined by the rf system,  $z_f$  will depend on parameters of both rf system and dispersive region. On the contrary,  $\delta_f$  depends only on parameters of the rf system applied to the initial condition.

As a final step, in order to characterize the electron beam before and after compression it is useful to introduce the standard deviations of the spatial and momentum distribution, giving the final bunch length and the correlated energy spread. Then, [14] and [15] transform into:

$$\sigma_{z,f} = \sqrt{R_{56}^2\sigma_{\delta,i}^2 + T_{566}^2\sigma_{\delta,i}^2 + (1 + \alpha_{rf}R_{56})^2\sigma_{z,i}^2 + (\alpha_{rf}^2 T_{566} + \frac{1}{2}R_{56}\beta_{rf})^2\sigma_{z,i}^2 + 4\alpha_{rf}^2 T_{566}^2\sigma_{z,i}^2\sigma_{\delta,i}^2} \quad [16]$$

$$\sigma_{\delta,f} = \sqrt{\sigma_{\delta,i}^2 + \alpha_{rf}^2\sigma_{z,i}^2 + \frac{1}{4}\beta_{rf}^2\sigma_{z,i}^2} \quad [17]$$

Eqs.[16] and [17] provide analytical solution for rms bunch length and rms correlated energy spread after compression, respectively, including beam transport up to the 2<sup>nd</sup> order both in the rf system and in the dispersive region.

Neglecting the 2<sup>nd</sup> order terms in [16] and the coupling terms between acceleration and dispersive motion (linear approximation), it is straightforward to estimate the required  $R_{56}$  for a given initial energy chirp and a desired final bunch length:

$$R_{56} \approx \frac{\sqrt{|\sigma_{z,i}^2 - \sigma_{z,f}^2|}}{\sigma_{\delta,i}^2} \quad [18]$$

### Compensating chromatic aberration using sextupoles

We report here some considerations about nonlinear optics in a chicane, in particular on the compensation of the chromatic aberration by mean of additional nonlinear magnetic field, i.e. sextupoles in the dispersive region.

Sextupolar magnetic field with normal reference to Cartesian coordinates is defined by<sup>2</sup>:

$$\begin{aligned} B_x &= 2m(x^2 - y^2) \\ B_y &= mxy \end{aligned} \quad [19]$$

where the parameter  $m$  is the sextupolar strength:

$$m = \frac{1}{2} \left( \frac{d^2 B_y}{dx^2} \right)_{x=0} \quad [20]$$

According to expression in [9], the sextupolar field may be expanded as follows:

$$B_x \cong 2m \left\{ \underbrace{2x_\beta \eta_x \delta + \eta_x^2 \delta^2}_{\text{Chromatic aberration}} + \underbrace{(x_\beta^2 - y_\beta^2)}_{\text{Geometric aberration}} \right\} + O(3)$$

[21]

$$B_y \cong m \left\{ \underbrace{x_\beta \eta_y \delta + y_\beta \eta_x \delta}_{\text{Chromatic aberration}} + \underbrace{x_\beta y_\beta}_{\text{Geometric aberration}} \right\} + O(3)$$

Geometric aberrations are a collateral effect of magnetic symmetry of a dipole and a sextupole. They can be made to vanish in a periodical structure, as an example if more than 4 identical cells with  $2\pi$  betatron phase advance are used. This implies requirement for quadrupole focusing.

Chromatic aberrations are an intrinsic effect of sextupolar focusing components and they can be made to vanish with at least one additional sextupole per plane that is a focusing and a defocusing sextupole per cell.

For these reasons, a 2<sup>nd</sup> order achromatic lattice has to include dipoles (for an arc or a chicane), quadrupoles for betatron focusing and sextupoles for chromatic corrections.

[21] contains explicit information about the field dependence on particle coordinates, dispersion and energy spread. The corresponding matrix notation for a sextupole in thin lens approximation is<sup>3</sup>:

$$M_{sext} = \begin{pmatrix} \cos \varphi & \left( \frac{1}{\sqrt{\bar{m}}} \right) \sin \varphi & 0 \\ -\sqrt{\bar{m}} \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad [22]$$

With reference to Figure 2, we can calculate the particle transport through a sextupole:

$$\begin{aligned} x_2 &= x_1 \cos \varphi + x_1' \frac{1}{\sqrt{\bar{m}}} \sin \varphi \\ x_2' &= -x_1 \sqrt{\bar{m}} \sin \varphi + x_1' \cos \varphi \end{aligned} \quad [23]$$

where phase  $\varphi \equiv l \sqrt{|m \eta_x \delta|}$  and normalized strength  $\bar{m} \equiv m \eta_x \delta$  are defined. Longitudinal coordinate of motion  $s$  of the particle when passing through a sextupole is then easily calculated:

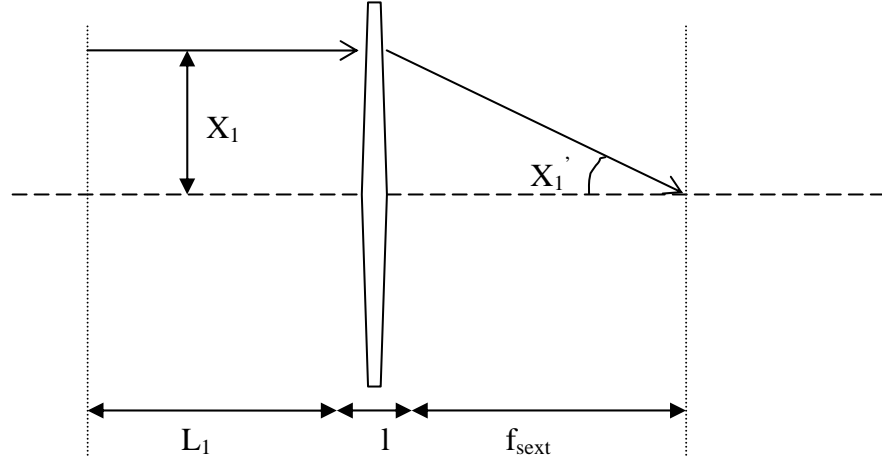
$$s = L_1 + \frac{f_{sext}}{\cos(x_2')} = L_1 + \frac{1}{m l^2 \cos(x_2')} \quad [24]$$

Finally, the components of the transport matrix up to 2<sup>nd</sup> order are calculated:

$$\begin{aligned} R_{56,sext} &= \left( \frac{ds}{d\delta} \right)_{\delta=0} = -\frac{\eta_x}{2} \cdot \frac{tg(x_1')}{\cos(x_1')} \cdot \{x_1 + lx_1'\} = f(\eta_x, x_1, x_1') \\ T_{566,sext} &= \left( \frac{d^2s}{d\delta^2} \right)_{\delta=0} \propto m \cdot \eta_x^2 \cdot g(x_1, x_1') \end{aligned} \quad [25]$$

From [24] it follows that  $T_{566}$  contribution to beam transport in a chicane can be compensated through sextupoles, using the linear proportionality between the  $T_{566,sext}$  term to be introduced and the sextupole strength. Nevertheless, [21] shows how sextupoles contribute with other detrimental effects to the transverse dynamics. In particular, chromatic and geometric aberrations increase the transverse emittances and

roll angles (in case of misalignment) may couple x and y plane (skew focusing). Thus, strong tolerances on magnetic field quality and on elements alignment are required.



**Figure 2.** A particle passing through a focusing lens of length  $l$ ;  $f_{\text{sext}}$  is (by definition) the focusing length associated to the element.

### Linearizing longitudinal phase space using a harmonic cavity

[16] includes nonlinear components contributing to bunch compression up to 2<sup>nd</sup> order in particle coordinates that is the rf curvature ( $\beta_{\text{rf}}$  term) and the chromatic aberration ( $T_{566}$  term). Below, we illustrate some scenarios in the ideal case of maximum compression, trying to find a solution for compensating the 2<sup>nd</sup> order particle dynamics, thus to linearize the longitudinal phase space. This operation is performed without introducing additional  $T_{566}$  term (no additional sextupoles) but just looking for an appropriate set up of phases in the booster linac. The electron beam so prepared in the linac will pass through the chicane and the 2<sup>nd</sup> order terms in [16] in principle will vanish.

Depending on what kind of bunch compressor we have, it is possible to use  $T_{566}$  to cancel against rf curvature (see, [16]), but this can only be done and still accelerating the beam if  $R_{56}$  has reversed sign with respect to a normal chicane (i.e.,  $R_{56}$  for a normal chicane is negative in this paper). It is possible to get positive  $R_{56}$  if using a FODO-cell arc (or similar) for compressor, but this is usually a longer system with more magnets and not transversely achromatic for all energies, as is a chicane.

In general, we can say all dispersive sections with no net bending angle have always  $R_{56} < 0$  (according to our convention), while arcs with net bending angle have always  $R_{56} > 0$ .

[16] states that  $\sigma_{z,f}$  reaches a minimum for:

$$(1 + \alpha R) = 0 \quad (\text{condition of maximum compression}) \quad [26]$$

and

$$(\alpha^2 T + 1/2 \cdot \beta R) = 0 \quad (\text{condition of phase space linearization}) \quad [27]$$

Conventionally in this paper we assume for a normal chicane  $R < 0$ , thus from [8] results  $T > 0$ . Since  $\alpha^2 > 0$ , linearization requires  $\beta \geq 0$  that is  $\cos\phi \leq 0$ . The latter condition means the rf system is decelerating the electron beam.

Different results can be obtained when including a more detailed description of the rf system with two or more distinct accelerating sections. In the following, three cases are considered and [26] and [27] applied to find a satisfactory solution for both bunch compression and compensation of nonlinearities in the longitudinal dynamics.

*Case A:* one rf section followed by a chicane (RF1 + DS).

$$\begin{aligned} \alpha^2 T + \frac{1}{2} \beta R = 0 \\ \bar{R} \equiv -\frac{1}{\alpha} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \text{tg}(\varphi_{rf}) &= -\left(\frac{\bar{R}}{3\tilde{\lambda}_{rf}}\right) \\ U_{rf} &= E_i \cdot \frac{\left(\frac{\tilde{\lambda}_{rf}}{R}\right)}{1+3\left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2} \cdot \sqrt{1+9\left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2} \end{aligned} \quad [28]$$

In this case RF1 decelerates and no energy gain is present in the linac. A different magnetic lattice must be chosen (an arc for example) with a positive  $R_{56}$ ; alternatively, at least one more section must be added to the booster linac as one more degree of freedom for the linearization.

*Case B:* two rf sections followed by a chicane (RF1 + RF2 + DS).

$$(\alpha_1 + \alpha_2)^2 T + \frac{1}{2} (\beta_1 + \beta_2) R = 0$$

$$\bar{R} \equiv -\frac{1}{(\alpha_1 + \alpha_2)}$$

$$\begin{aligned} \text{tg}(\varphi_2) &= -\frac{E_i \left(\frac{\tilde{\lambda}_{rf}}{R}\right) - \bar{G}_1 \left[1 + 3\left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2\right]}{3E_i \left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2 + \bar{G}_2 \left[1 + 3\left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2\right]} \\ \Rightarrow \quad U_{rf} &= \sqrt{\left\{ \frac{E_i \left(\frac{\tilde{\lambda}_{rf}}{R}\right)}{\left[1 + 3\left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2\right]} - \bar{G}_1 \right\}^2 + \left\{ \frac{3E_i \left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2}{\left[1 + 3\left(\frac{\tilde{\lambda}_{rf}}{R}\right)^2\right]} + \bar{G}_2 \right\}^2} \end{aligned} \quad [29]$$

where  $\bar{G}_1 = U_1 \cos\varphi_1$  and  $\bar{G}_2 = U_1 \sin\varphi_1$ .

In this case, compensation of the rf curvature and  $T_{566}$  is provided by RF1, but RF2 decelerates the beam and  $U_2$  and  $\phi_2$  are uniquely determined by the choice of  $U_1$  and  $\phi_1$ .

Case C: two rf sections followed by a high harmonic cavity and a chicane (RF1 + RF2 + RFX + DS).

$$\begin{aligned}
 & (\alpha_1 + \alpha_2)^2 T + \frac{1}{2}(\beta_1 + \beta_2 + \beta_x)R = 0 \\
 & \bar{R} \equiv -\frac{1}{(\alpha_1 + \alpha_2)} \quad \Rightarrow \quad \varphi_x = \pi \\
 & U_x = \frac{E_f \left[ 1 + 3 \left( \frac{\lambda_{rf}}{\bar{R}} \right)^2 \right] - E_i}{\left( \frac{\lambda_{rf}}{\lambda_x} \right)^2 - 1} \quad [30]
 \end{aligned}$$

This layout is quite flexible as for management of the longitudinal phase space; it allows beam acceleration with two high gradients accelerating sections, while phase space linearization is performed by the low voltage harmonic cavity, set to net deceleration. [30] show the energy loss due to the harmonic cavity is minimized by operating the cavity at a frequency which is a high harmonic of the linac frequency (higher the frequency, lower the peak voltage)<sup>1</sup>.

## Conclusions

The electron beam dynamics in a magnetic compressor has been studied. The theoretical formulas expanded up to 2<sup>nd</sup> order in the particles coordinates have been explicitated and applied to some specific cases.

The 2<sup>nd</sup> order beam dynamics study includes: i) the rf curvature, which comes from the beam acceleration in the booster linac just before the chicane; ii) the 2<sup>nd</sup> order momentum compaction (T<sub>566</sub> term in the transport matrix); schemes of compensation of this term have been considered for the specific case of a pure chicane made of rectangular dipoles.

It has been considered the possibility of correction of the chromatic aberration in the beam compression using a symmetric couple of sextupoles inserted in the chicane. The linearization of the longitudinal phase space is possible but adding also large transverse aberrations. In addition to the T<sub>566</sub> compensation, there are also three transverse terms to balance: a) the geometric aberrations (~x<sup>2</sup>), b) the chromatic aberrations (~x·Δp/p) and c) the 2<sup>nd</sup> order dispersion ([Δp/p]<sup>2</sup>). The last of these is the most significant when sextupoles are placed at high dispersion. In addition to these aberrations, the alignment of sextupoles becomes a strong issue, too, since horizontal misalignments will generate anomalous horizontal dispersion, corrupting the horizontal emittance; while vertical misalignments generate vertical dispersion, corrupting the vertical emittance.

An alternative solution for linearization of the longitudinal phase space has been found using a high harmonic cavity before the chicane. The linearization is equivalent to that obtained by the sextupoles. With the cavity, deceleration of the electron beam required for nonlinearities compensation is minimized by the high frequency of the structure and the effects of its strong wakefields are limited by the relative small length of the device (generally, not longer than 1 m).

Analytical formulas up to 2<sup>nd</sup> order have been applied to a case which foresees an accelerating linac (characterized by sections with high accelerating gradient), followed by the decelerating (low voltage) high harmonic cavity and finally a pure chicane of rectangular magnets.

## **References**

<sup>1</sup> I. V. Bazarov, ERL 02-2, November 2001

<sup>2</sup> E. Wilson, CAS – 5<sup>th</sup> General Accelerator Physics Course, CERN 94-01, Vol. I, p.239, Geneva 1994.

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