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ELEGANT CALCULATIONS OF CSR INDUCED MICROBUNCHING IN THE BENCHMARK BUNCH COMPRESSOR

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Abstract

Elegant simulations of electron transport, including CSR, have been carried out on the high energy benchmark bunch-compressor to study the numerical convergence of the problem as well as to investigate the importance of CSR micro-bunching.

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1. INTRODUCTION

Micro-bunching due to CSR has been predicted theoretically, see for example [1] and observed experimentally on storage rings, for example [2]. Any disruption of the high charge bunches required for FEL operation due to CSR is a potential problem, and hence it is important to be able to predict the effect of CSR. However, a large number of macro-particles must be used in simulations in order for the effects of micro-bunching to be modelled and this is only possible at present using 1D simulations of the CSR. ELEGANT [3] uses the linear current density model of Saldin et al [4] to evaluate CSR, and can be run with millions of macro-particles. Bunching has been observed; however, there is concern that some of the structure seen in the resulting bunch profiles is due to numerical noise.

Benchmarks were devised to look at the calculation of CSR through a bunch compressor [5]. The bunch-compressor is the same for all cases, but the bunch properties change, in particular the bunch charge and energy. As the effect of CSR does not depend on bunch energy for highly relativistic electrons, but increases with bunch charge, the high charge bunch case, appropriate for the LCLS, has been chosen for this study. In this case the uncorrelated energy spread of 10 keV is relatively smaller with respect to the electron energy, and will not wipe out the microbunching so easily.

The bunch compressor is a simple 4 dipole chicane, with projected magnet lengths of 0.5 m. The inner two magnets are separated by 1m, the outer magnets by 5m drifts. The bending angle is 2.77 deg. The R_{56} term is -25mm, T_{566} is +27.5mm. The ELEGANT file used for the lattice, adapted from P Emma's file, is given in the Appendix.

The main numerical factors which are important in the numerical accuracy of these calculations are the number of macroparticles and the number of bins used to calculate the linear current needed for the CSR calculations. The more macroparticles the closer one gets to the real case of single electrons. The larger the number of particles per histogram bin for the current density the smaller the statistical noise in the current, but if there are too few bins then any higher frequency structure is smoothed out. This is particularly important for the modulated bunch simulations.

The first set of calculations uses a pure Gaussian bunch, the second imposes an oscillation on top of the Gaussian profile. The results for the Gaussian bunch are given in section 2 and for the modulated bunch in section 3. The conclusions are presented in section 4.

2. GAUSSIAN BUNCH

Table 1. Benchmark parameters for the input bunch

Electron energy	5Gev
Bunch charge	1nC
Incoherent rms energy spread	10keV = 0.002%
Initial rms bunch length	200 μ m
Final rms bunch length	20 μ m
Initial normalised emittance	1.0 mm mrad
Initial beta functions	$\beta(x)=40$ m $\beta(y)=13$ m
Initial alpha functions	$\alpha(x)=2.6$, $\alpha(y)=1.0$
Linear chirp	+36 m^{-1}
Total initial energy spread	0.72%

The benchmark parameters for the bunches are given in Table 1. These parameters yield the rms values of the bunch distributions for the input bunch parameters given in Table 2. Halton sequences were used to generate the distributions, giving a "quiet" start. It was found that the final rms values of the bunch distributions, also given in Table 2, do not depend on the numerical parameters used in ELEGANT. The origin of the small energy loss in the non-CSR case is not known – this case used the CSRCSBEND element

in ELEGANT with CSR=0. The variation of the bunch length, energy spread and emittance are shown in Figure 1. The bunch is being very slightly overcompressed.

Table 2. Input and final bunch parameters

Bunch	$\sigma(x)$ (μm)	$\sigma(x')$ * 10^6	$\sigma(y)$ (μm)	$\sigma(y')$ * 10^6	$\sigma(s)$ (μm)	$\sigma(\delta)$ %	Average momentum	Energy loss %
Input	63.9	4.45	36.4	3.97	200	0.720	9784.736	
Final no CSR	24.1	4.45	41.3	3.98	20.17	0.720	9784.414	0.003%
Final with CSR	24.9	6.78	41.3	3.98	20.4	0.714	9780.093	0.047%

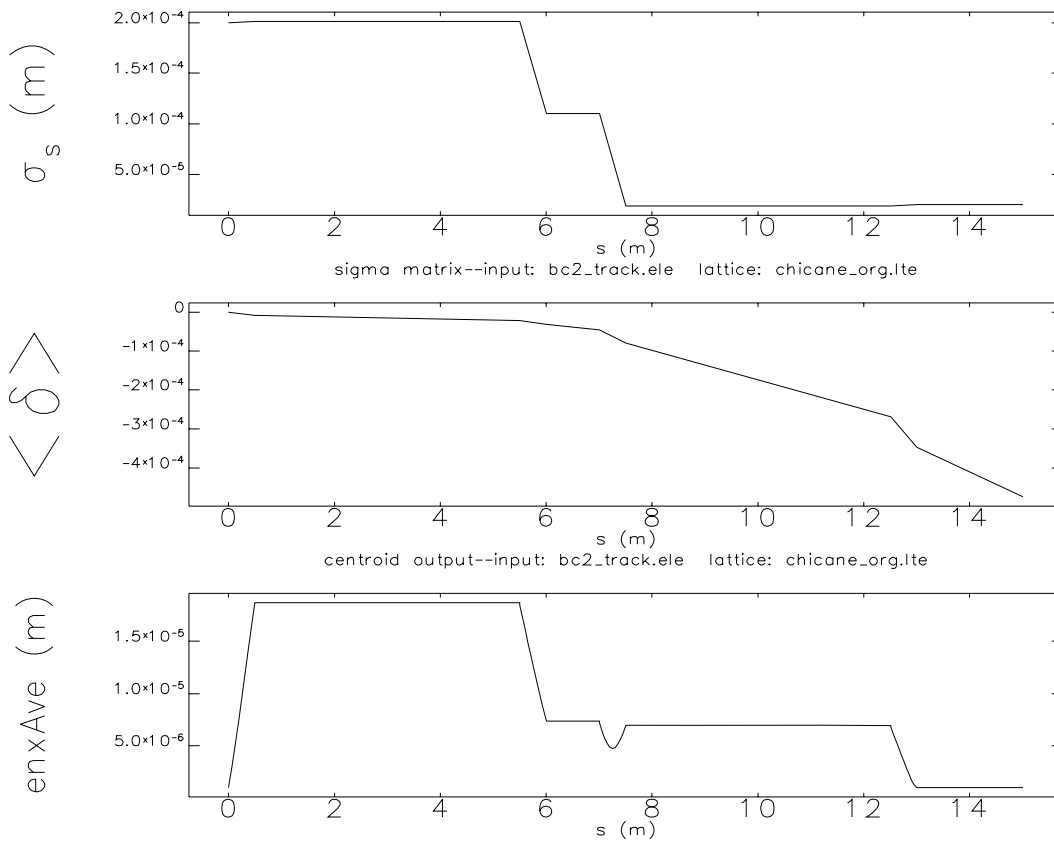


Figure 1. rms bunch length (top), average relative energy loss (middle) and slice emittance (bottom) through the bunch compressor.

The amount of structure in the bunch current density was investigated as a function of the number of macro-particles and bin size used to calculate the CSR. The results are presented as sums over the Power Spectral Densities (PSD) in Table 3. The PSDs themselves were calculated using different bins sizes for the histogram of the current to ensure that no additional smoothing was being carried out due to the analysis. It is seen that the amount of bunching decreases with increasing number of macro-particles for a given number of CSR bins, and increases with the number of CSR bins for a given number of macro-particles, suggested that the bunching is in part due to statistical noise in calculating the linear current density. The ELEGANT manual warns that care has to be taken not to have too many bins for this very reason.

Table 3. Power in PSD for different numbers of macro-particles and bins used to calculate the CSR.

No of macroparticles	No of CSR bins	No of particles/bin	Σ PSD ($2 \cdot 10^{13} - 4 \cdot 10^{14}$) Hz	Σ PSD ($2 \cdot 10^{13} - 8 \cdot 10^{14}$) Hz
0.2M	500	400	$1.5 \cdot 10^4$	$1.5 \cdot 10^4$
0.5M	500	1000	$5.4 \cdot 10^3$	$5.5 \cdot 10^3$
1M	500	2000	$2.3 \cdot 10^3$	$2.4 \cdot 10^3$
	1000	1000	$1.4 \cdot 10^4$	$1.5 \cdot 10^4$
2M	500	4000	$1.1 \cdot 10^3$	$1.2 \cdot 10^3$
	1000	2000	$6.9 \cdot 10^3$	
	2000	1000	$1.4 \cdot 10^4$	$1.7 \cdot 10^4$
3M	500	6000	$9.9 \cdot 10^2$	$1.0 \cdot 10^3$
1M	No CSR			$1.2 \cdot 10^2$

The transforms of the current densities for different numerical parameters are given in Figure 2. The effect of the binning can be seen in smoothing the data so that power is not seen at the higher frequencies for the cases with 500 bins (top 2 graphs). The highest sampling frequency in the final bunch (with 2000 bins) is roughly $5 \cdot 10^{15}$ Hz – this is important when looking at the modulated Gaussian bunches. The power also decreases as the number of particles/bin increases, and from these results we can see that most of the structure is numerical in origin and any CSR induced micro-bunching is of low amplitude. The profile for the smoothest final bunch calculated, ie with 3 million macroparticles and 500 CSR bins, is given in Figure 3 and a comparison of its Fourier transform with the case run with no CSR in Figure 4.

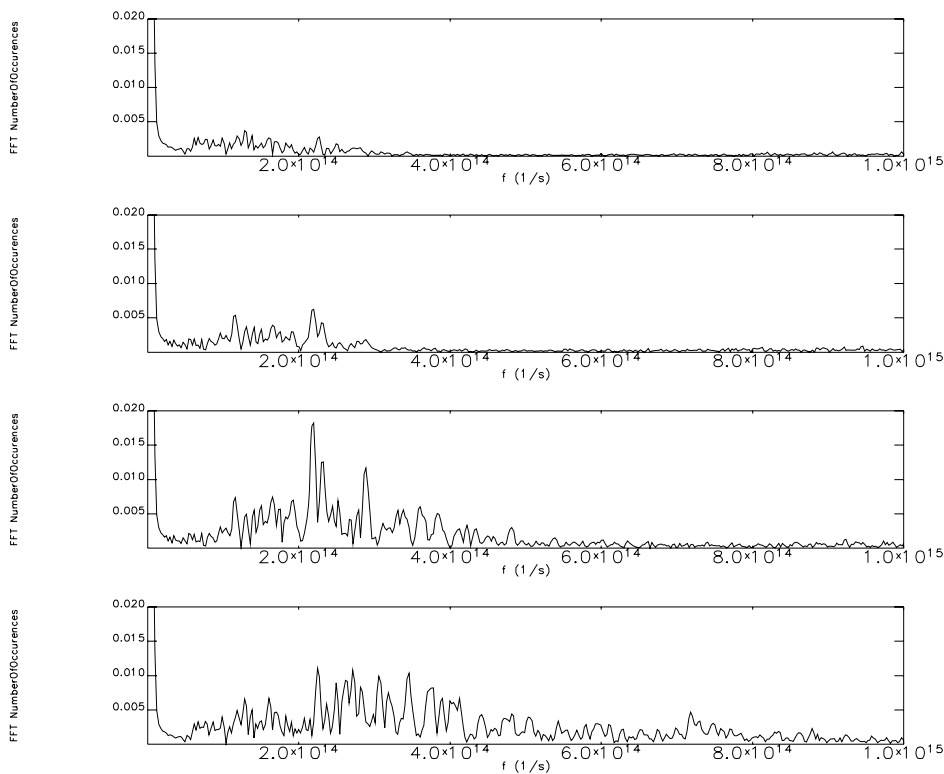


Figure 2. A comparison of the transforms of the final current density for, from top to bottom, 2 million macroparticles, 500 bins; 1 million macroparticles, 500 bins; 1million macro-particles 1000 bin; , 2 million macroparticles, 2000 bins.

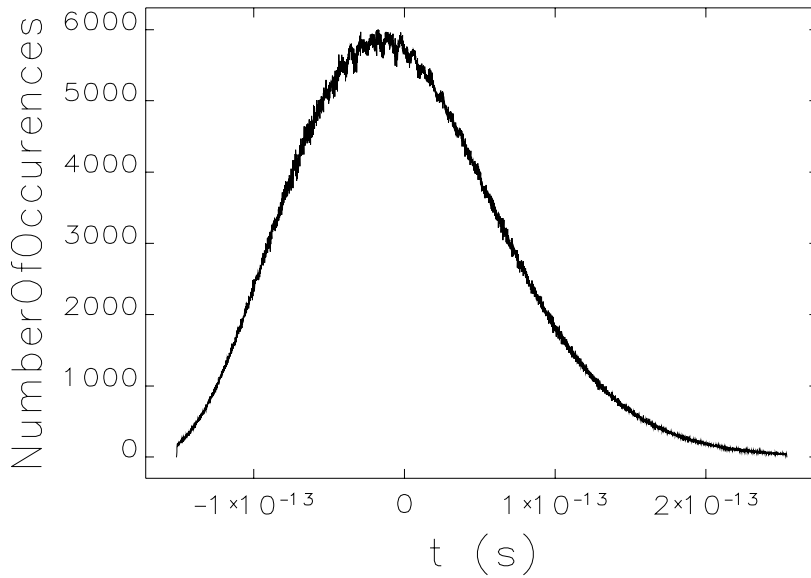


Figure 3. Final current density for the case with 3 million macro-particles and 500 bins.

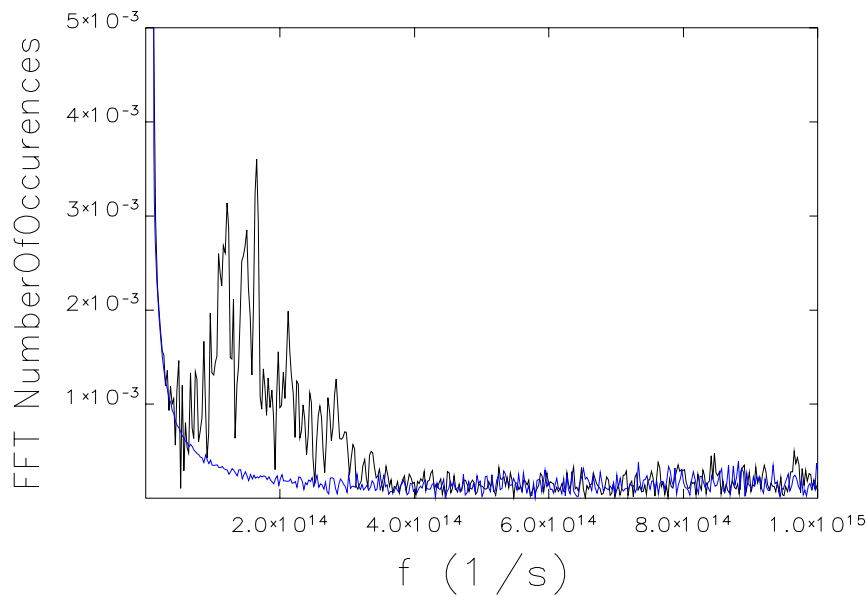


Figure 4. A comparison of the normalised Fourier transform of the final bunch profile, calculated with CSR (black) and without CSR (blue). 3 million macroparticles, and 500 bins were used in the case with CSR.

The modulation in the current density ‘disappears’ through the bunch compressor. For example, a comparison of the Fourier transform of the current density at the end of the third and fourth magnets is given in Figure 5. Note that the bunch length is very similar for both cases. The ‘disappearance’ of the modulation is due to the combination of the non-zero dispersion through the bunch compressor (Figure 6), and the increase in slice energy spread (Figure 7).

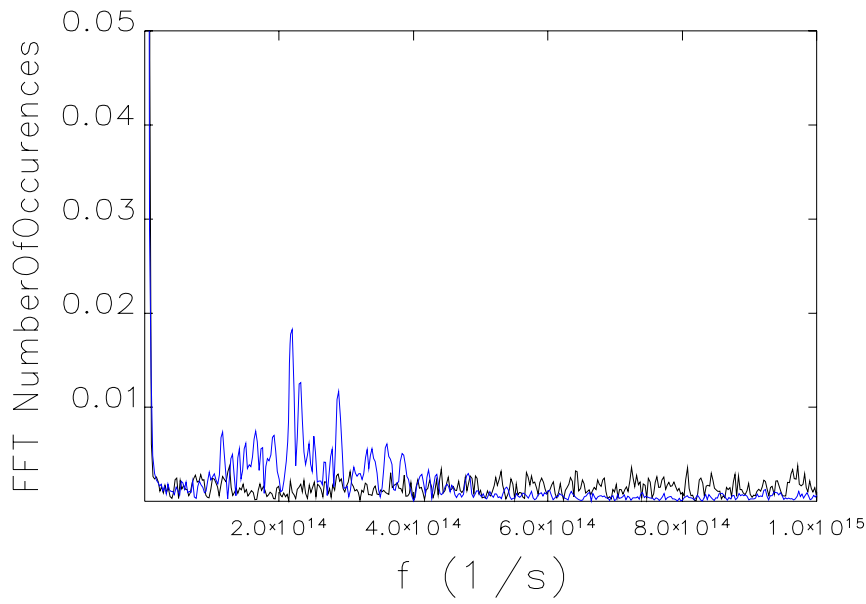


Figure 5. Comparison of the normalised Fourier transform of the current density at the end of the 3rd dipole (black) and the end of the 4th dipole (blue), for the case of 1 million macroparticles.

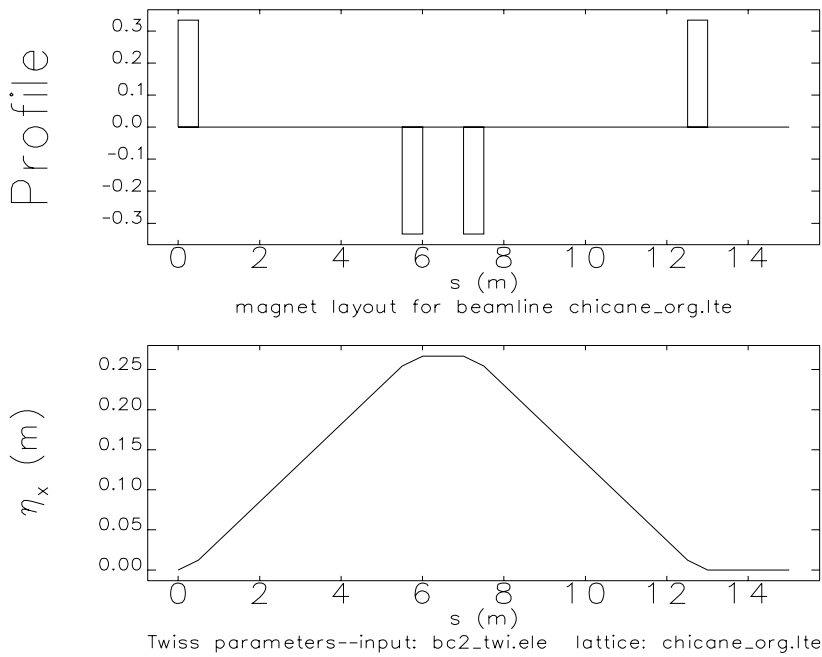


Figure 6. Top – magnet profile: Bottom - horizontal Dispersion through the bunch compressor

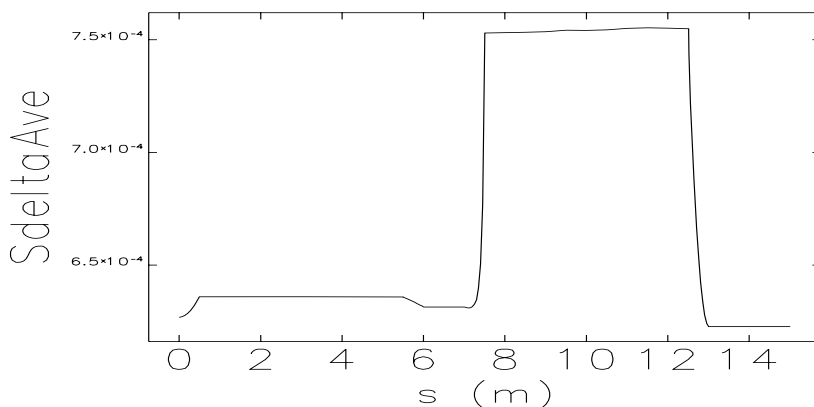


Figure 7. Average of the slice energy spread for the case of no CSR.

3. MODULATED BUNCH.

The bunch compressor is the same as for the previous section. This time the bunch has a Gaussian profile with a flat top inserted in the middle, the length of which has to be 20 times the modulation wavelength. The whole bunch is to be modulated, with a sinusoidal modulation of relative amplitude before compression of 0.1% and wavelength in the final bunch of 0.5, 1, 2, and 5 μm . The final bunch has to achieve a peak current of 6kA. As the compression is a factor 10, the initial modulation wavelength ranged from 5 to 50 μm . Initial uncorrelated energy spreads of zero (set to 10^{-7}) and $4 * 10^{-5}$ and normalised horizontal emittances of 1 and 10 nm rad were used. The vertical emittance was 1 nm rad.

Bunches with Gaussian sides of different widths were tried until the correct final peak current was obtained. Note that the requirement of 20 wavelengths on the flat top meant that for the longest wavelength modulation, the initial bunch is longer than for the other cases and the peak current was not quite reached. The bunches given in Table 4 were used for the numerical studies. The sums over the PSDs for the three shortest wavelengths avoid the initial peak in the PSD from fitting the overall bunch profile, however, the modulation frequency is within the initial peak for the longest wavelength.

Table 4. Input bunch parameters.

Initial Modulation wavelength (μm)	σ of Gaussian sides (μm)	σ (s) (μm)	σ (δ) %	Σ PSD
5	150	169	0.611	0.067
10	120	165	0.593	0.170
20	60	162	0.585	0.063
50	10	296	1.065	120 ^s

^sThe modulation frequency is within the envelope of frequencies which describe the bunch shape.

As in the non-modulated case, the rms values of the final bunch distributions were not dependent on the numerical settings in the code and are given in Table 5. Not all cases were run, as nothing new was being learned. The slight increase in final energy for the 50 μm modulation case is due to the increased bunch length. The numbers in Table 5 were obtained using 3 million macroparticles and 1000 bins for the CSR.

Table 5. Final bunch parameters

wavelength (μm)	ϵn nm rad	$\sigma(\delta)$ % initial	$\sigma(x)$ (μm)	$\sigma(x')$ $*10^6$	$\sigma(y)$ (μm)	$\sigma(y')$	$\sigma(s)$ (μm)	$\sigma(\delta)$ %	Average momentum	Energy loss %
5	1	$1*10^{-7}$	69.1	5.87	51.3	2.61	17.2	0.60	9779.25	0.056
10	1	$1*10^{-7}$	69.5	5.47	50.6	2.55	16.7	0.59	9779.03	0.058
		$4*10^{-5}$	69.4	5.43	50.0	2.55	16.7	0.59	9779.04	0.058
	10	$1*10^{-7}$	218	7.34	50.6	2.55	16.7	0.59	9779.44	0.054
		$4*10^{-5}$	218	7.34	50.6	2.55	16.7	0.59	9779.45	0.054
20	1	$1*10^{-7}$	69.7	4.58	50.0	2.55	16.5	0.58	9778.98	0.059
		$4*10^{-5}$	69.7	4.59	50.6	2.55	16.5	0.58	9779.00	0.059
	10	$1*10^{-7}$	218	7.06	50.6	2.55	16.5	0.58	9779.47	0.054
50	1	$1*10^{-7}$	69.2	3.08	51.5	2.62	29.7	1.1	9781.28	0.035
		$4*10^{-5}$	69.2	3.08	51.5	2.62	29.8	1.1	9781.31	0.035
	10	$1*10^{-7}$	217	5.69	50.6	2.55	29.7	1.1	9781.55	0.033

3.1 10 μm modulation results.

The effect of changing the numerical parameters was investigated for the 10 μm case, with normalised emittance of one and very small uncorrelated energy spread. If one assumes that the modulation wavelength decreases with the bunch compression, one would expect a modulation of 1 μm in the final bunch. The total bunch length is about 100 μm , and in order to see any oscillations of around 1 μm ($3 * 10^{14}$ Hz) at least 400 bins in the current histograms should be used. The results of the non-modulated bunch imply that in fact about 1000 bins are required to avoid smoothing at these frequencies.

The Fourier transform of the final current density for the cases of 3 million macroparticles, 1000 bins and 4 million macroparticles, 2000 bins are shown in Figure 8. There is no obvious peak at $3*10^{14}$ Hz, but a broad range of frequencies is seen. There is slightly more power in the frequencies below $5*10^{14}$ Hz in the case with 2000 CSR bins – this could be an increased noise component as the number of particles per CSR bin is less. The sampling frequency with 2000 bins is $6*10^{15}$ Hz, so it is unlikely that the increased power at higher frequencies is entirely noise related.

For comparison, the Fourier Transforms using 4 million particles, with the same bunch profile but with no-modulation, and with the modulation but no CSR are given in Figure 9. The effect of the 10 μm modulation is to give an increase in the power at all frequencies.

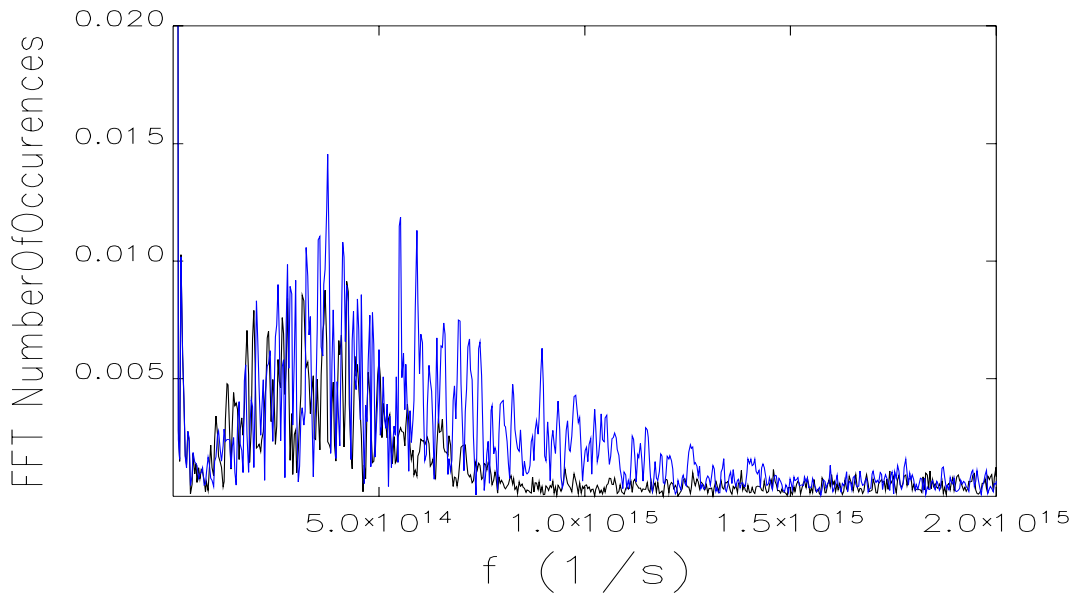


Figure 8. Normalised Fourier Transform of the final current density for the 10 μm initial modulation. Black – 3 million macroparticles, 1000 CSR bins, Blue – 4 million macroparticles, 2000 CSR bins.

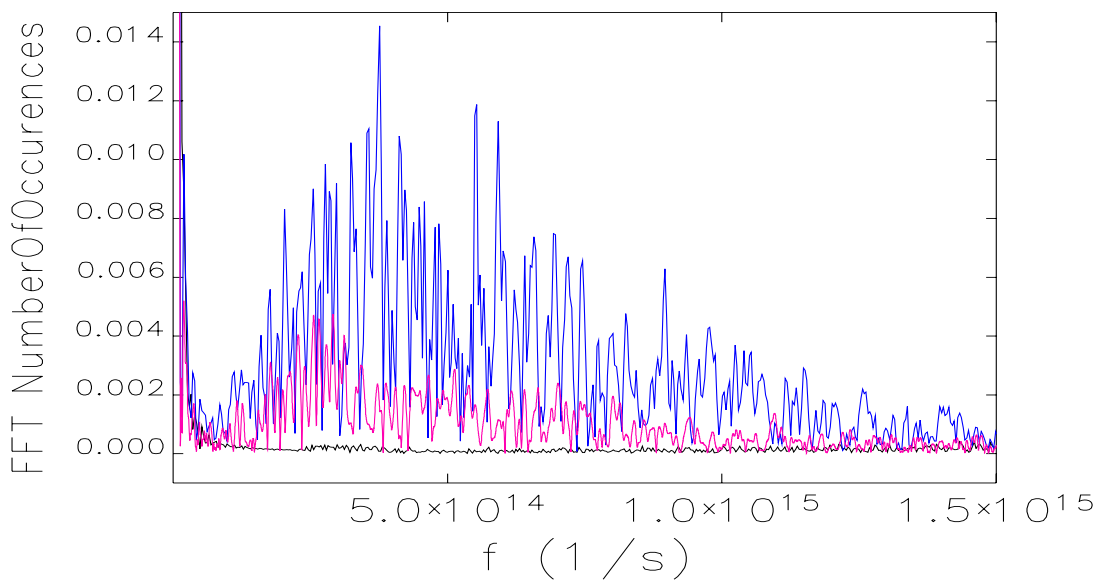


Figure 9 Normalised Fourier Transform of the final current density for Black – no CSR, Pink – CSR but no modulation, Blue 10 μm modulation. All run with 4 million macroparticles.

3.2 Comparison of results for different modulations wavelengths

The Fourier transform of the final current density for the 10 μm case is compared with that of the 5 μm modulation in Figure 10 and the 20 μm modulation Figure 11. There does not seem to be a significant

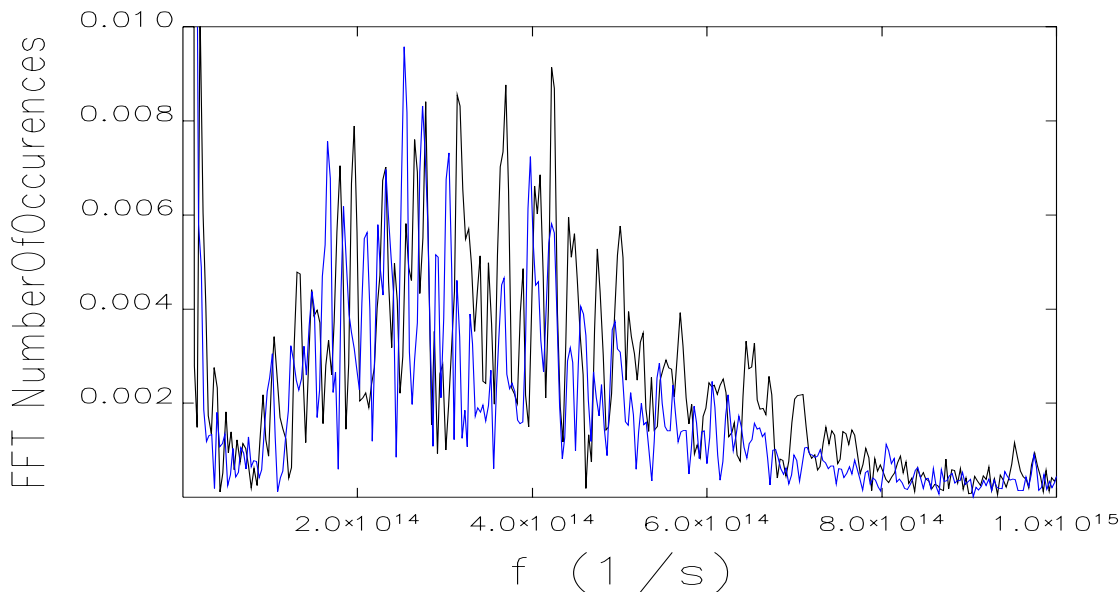


Figure 10. Comparison of the Fourier transform of the final current density for 10 μm modulation (black) and 5 μm (blue)

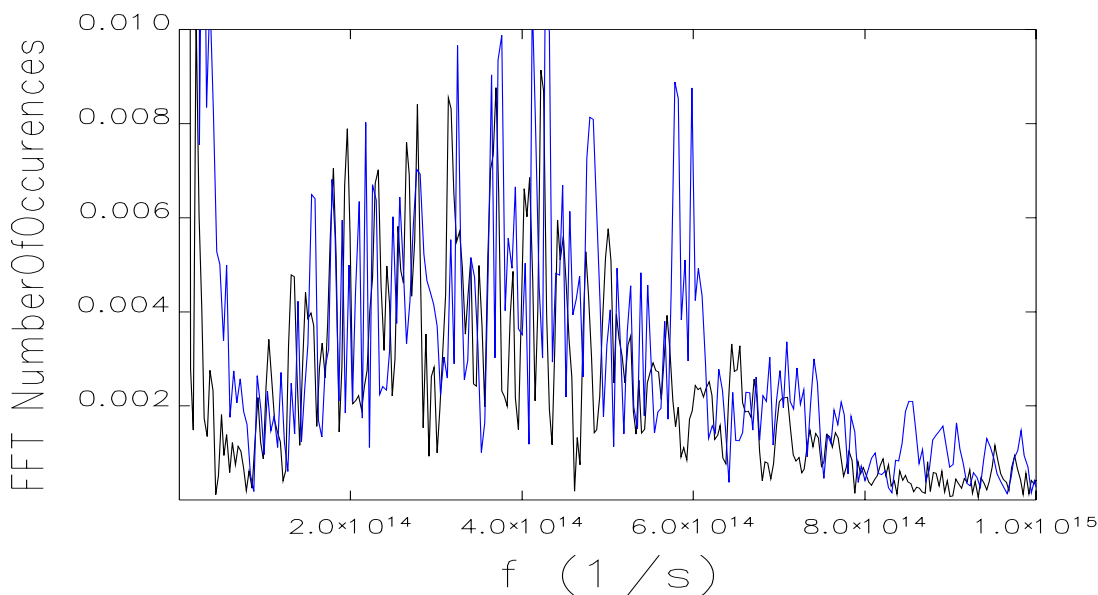


Figure 11. Comparison of the Fourier transform of the final current density for 10 μm modulation (black) and 20 μm (blue)

difference in the shape of the transforms around the frequencies of interest, from $(1.5-6) \times 10^{14}$ Hz, corresponding to modulation wavelengths of 0.5 – 2 μm. However there is more power in the oscillations for

the 20 μm case. The bunch profile is very different for the 50 μm which makes it hard to meaningfully compare the results, as is shown in Figure 12.

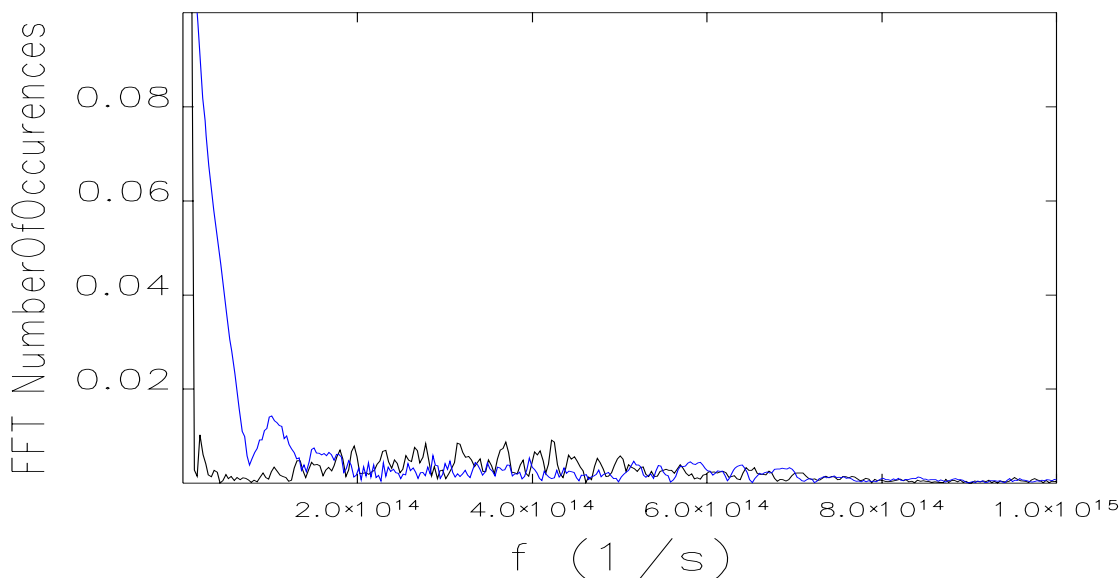


Figure 12 Comparison of the Fourier transform of the final current density for 10 μm modulation (black) and 50 μm (blue).

In error, the modulation amplitude for the benchmark was initially thought to be 5%, and some results were obtained for 5 μm and 10 μm modulations with this amplitude. It was found that the oscillation amplitude was significantly larger for the longer wavelength, reinforcing the possible trend in oscillation amplitude with modulation amplitude seen above.

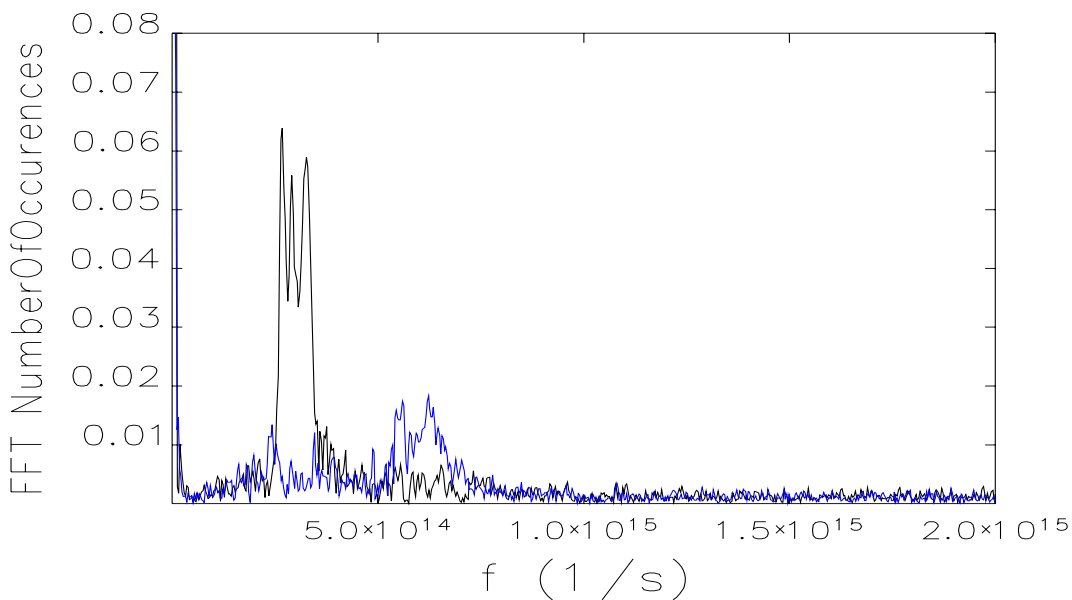


Figure 13 Comparison of the Fourier transform of the final current density for initial 10 μm modulation (black) and 5 μm (blue) with a 5% amplitude oscillation. Cases run with 2000 macroparticles and 2000 bins.

A comparison is given with the Fourier Transform for the 10 μm modulation case and a Gaussian bunch with a random distribution (Figure 14) whose input current density has the Fourier Transform shown in Figure 15. We see more enhancement of the power spectra for the lower frequencies ($1 - 2.5 \times 10^{14}$ Hz) than for frequencies $> 2.5 \times 10^{14}$ Hz (modulation wavelength $\sim 1.2 \mu\text{m}$), again suggesting that the gain is greater for the longer wavelengths. Note however, in the comparison between bunches of the same shape, one with a 10 μm oscillation and one with no oscillation, the relative gain in is not so large for the lower frequencies around 10^{14} Hz (pink curve Figure 9). The input spectrum is very quiet (Figure 16).

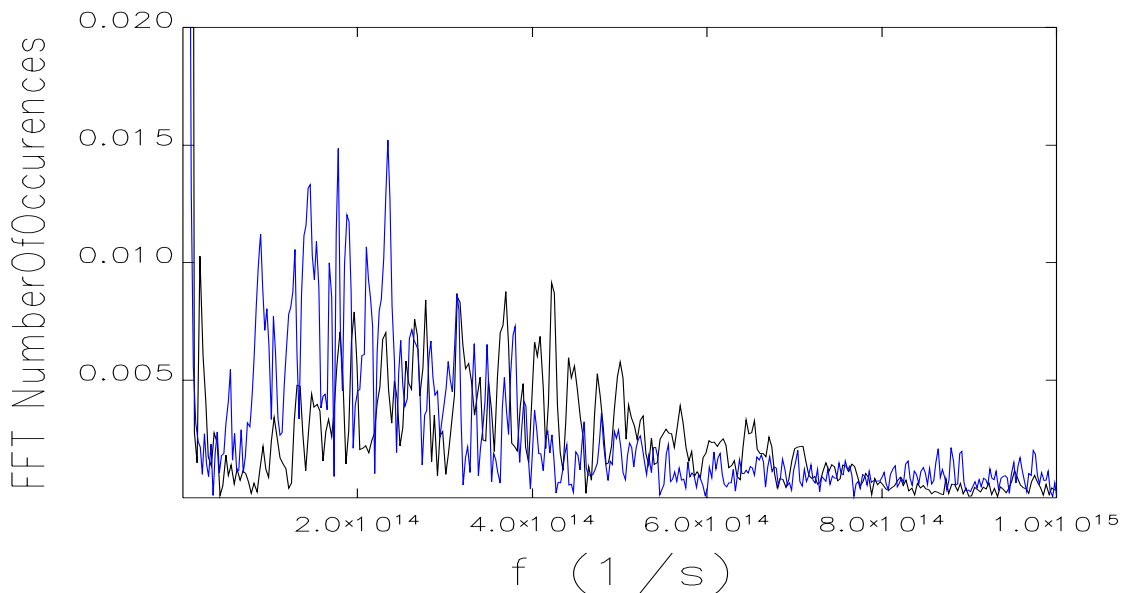


Figure 14. Comparison of the Fourier Transform of the current density for the final bunch for the 10 μm modulation case (Black) and a randomly distributed Gaussian bunch (Blue)

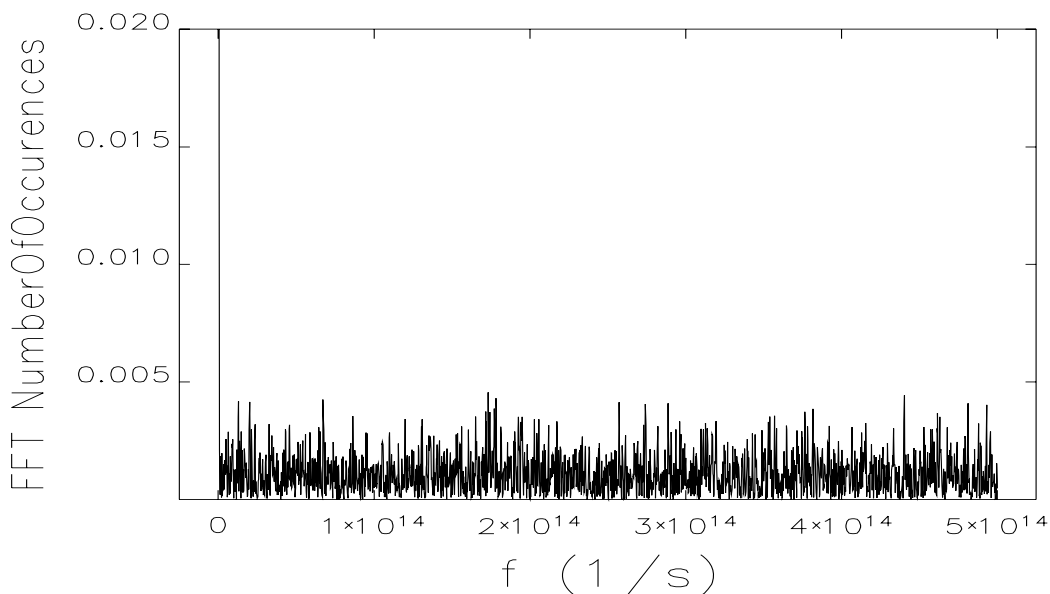


Figure 15. Fourier Transform of the current density of the input randomly distributed bunch.

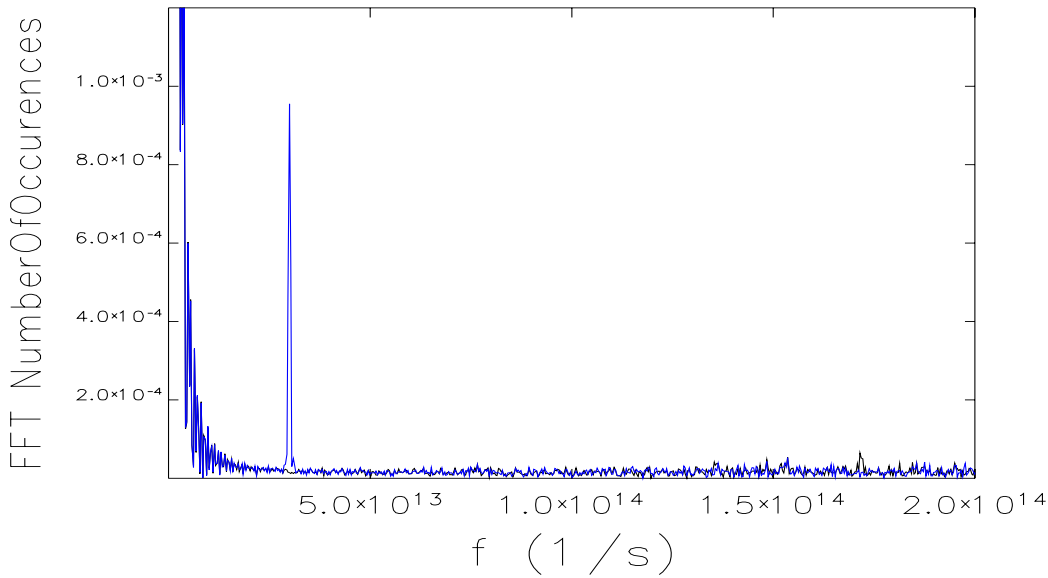


Figure 16 Fourier Transform of the current density of the input bunch with the same shape as the 10 μm modulated bunch, but with no modulation (black) and for the bunch with the 10 μm modulation (blue)

3.3 Effect of increasing the emittance and uncorrelated energy spread.

Increasing either the horizontal emittance or uncorrelated energy spread substantially reduces amount of micro-bunching, as can be seen for the 10 μm modulation case in Figure 17. The increased horizontal emittance in particular wipes out the higher frequencies whereas the effect of increased uncorrelated energy spread seems to generate a noise spectrum.

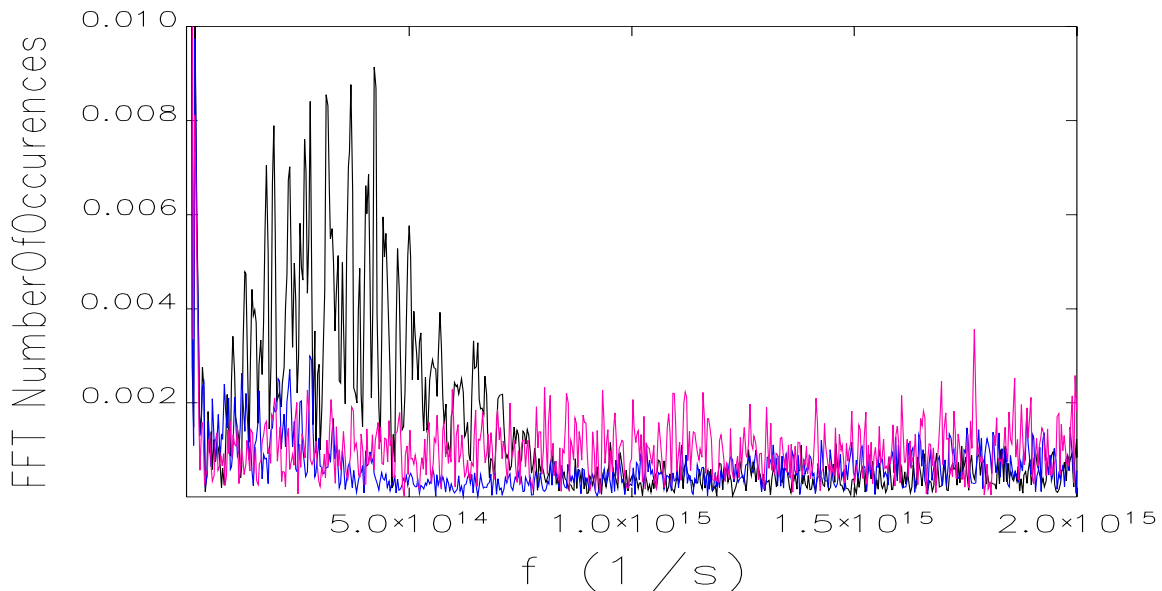


Figure 17 Normalised Fourier Transform of the final current density for 10 μm modulation case, run with 3 million macroparticles, 1000 CSR bins. Black – normalised emittance=1, Blue – normalised emittance=10, Pink – normalised emittance=1, rms uncorrelated energy spread $4 * 10^{-5}$.

In order to try to quantify the amount of micro-bunching, the PSDs have been summed from $(1-8) * 10^{14}$ Hz which covers most of the range of increased power due to micro-bunching (see Figure 9) except for the 50 μ m oscillation, and for the range $(0.3-8) * 10^{14}$ Hz which includes the range of increased power for the 50 μ m oscillation but also includes part of the low frequency peak. The results given in Table 6.

Table 6. Sum over Power Spectral Densities.

wavelength (μ m)	ϵ n nm rad	σ (δ) % initial	Σ PSD (1-8) * 10^{14} Hz	Σ PSD (0.3-8) * 10^{14} Hz
5	1	$1 * 10^{-7}$	$1.20 * 10^4$	$1.22 * 10^4$
10	1	$1 * 10^{-7}$	$1.49 * 10^4$	$1.53 * 10^4$
		$4 * 10^{-5}$	$1.45 * 10^3$	$1.57 * 10^3$
	10	$1 * 10^{-7}$	$9.4 * 10^2$	$1.19 * 10^3$
		$4 * 10^{-5}$	$1.45 * 10^3$	$1.65 * 10^3$
20	1	$1 * 10^{-7}$	$2.52 * 10^4$	$2.86 * 10^4$
		$4 * 10^{-5}$	$1.79 * 10^3$	$3.23 * 10^3$
	10	$1 * 10^{-7}$	$1.54 * 10^3$	$3.45 * 10^3$
50	1	$1 * 10^{-7}$	$1.26 * 10^4$	$1.10 * 10^5$
		$4 * 10^{-5}$	$1.27 * 10^3$	$2.09 * 10^4$
	10	$1 * 10^{-7}$	$1.13 * 10^3$	$8.84 * 10^4$

Calculation on the micro-bunching gain for a similar bunch compressor to the benchmark design have been carried out by Heifets et al [6]. Their Figure 3 shows the amplification as a function of the initial wavelength of the modulation. For their case of smallest uncorrelated energy spread of $3 * 10^{-6}$ and normalised emittance of 1 mm mrad, the gain curve peaks at nearly 3 around 20 μ m, and had a values of 1.8, 2.1 and 2 for modulation wavelengths of 5, 10 and 50 μ m respectively, consistent with the above simulations. For their higher energy spread case of $3 * 10^{-5}$, the gain is zero for wavelengths less than about 12 μ m, and peaks around 40 μ m. In this case, the gain at 20 and 50 μ m is 0.8 and 1.6 respectively. The smallest decrease in gain with increased energy spread occurs at 50 μ m, again consistent with the above results.

4. SUMMARY

The results from ELEGANT simulation of the benchmark bunch compressor show that a large number of macroparticles, at least a few million, are required to simulate micro-bunching. Care has to be taken in setting the number of bins used to calculate CSR, especially if high frequency features are to be studied. As expected, even a small uncorrelated energy spread of $4 * 10^{-1}$ decreases the amount of micro-bunching considerably, as does an increase in horizontal emittance. The behaviour of the microbunching gain is consistent with theoretical predictions.

References.

1. K Huang and K-J Kim, *Phys. Rev. ST Accel. Beams* **5**, (2002), 074401
2. M Abo-Bakr, J Feikes, K Holldack, P Kuske G Wuestefeld, EPAC03 Proceedings, 3023.
3. M Borland, *Phys. Rev. ST Accel. Beams* **4**, (2001), 070701
4. E L Saldin, E A Schneidmiller and M Yurkov, *Nucl Instr and Meth* **A398** (1997), 373
5. <http://www.desy.de/csr>
6. S Heifets, G Stupakov and S Krinsky *Phys. Rev. ST Accel. Beams* **5**, (2002), 064401

Appendix

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!TITLE,S="Medium-Length Single Chicane for LCLS BC2 at 5 GeV"
!-----
!04-OCT-01
!P. Emma
!-----
!
! Cb    : CONSTANT=1.0E10/CLIGHT ! energy to magnetic rigidity
! rad2deg : CONSTANT=180.0/PI    ! factor to take radians to degrees

! EBC2  := 5.000          ! BC2 energy (GeV)
! EMITXN := 1.000E-06 ! normalized horizontal emittance (m)
! EMITYN := 1.000E-06 ! normalized vertical emittance (m)
! BLENG  := 0.200E-03 ! bunch length (m)
! ESPRD  := 0.720E-02 ! energy spread (1)
! bX     := 40.0        ! twiss beta x (m)
! aX     := 2.6         ! twiss alpha x
! bY     := 13.0        ! twiss beta y (m)
! aY     := 1.0         ! twiss alpha y

! construct input beam matrix (assumes DY=DPY=0)

! BC2

! Brho := Cb*EBC2        ! beam rigidity at BC2 (kG-m)
! BB   := -16.120960797916 ! chicane bend field (kG)
! RB   := Brho/BB        ! chicane bend radius (m)
! AN   := ASIN(LB/RB)    ! full chicane bend angle (rad)
! LBS  := RB*AN          ! chicane bend path length (m)

! magnet-to-magnet path lengths

! LD   := 5.0           ! outer bend-to-bend "Z" distance (m)
! LDo  := LD/COS(AN)    ! outer bend-to-bend path length (m)
! LDi  := 1.0           ! inner bend-to-bend "Z" distance (m)

!bends:
!=====
B1: csrbend, angle=0.0483456, l=0.5000484, e1=0, e2=0.0483456, synch_rad=1, &
Nonlinear=1, n_kicks=51, integration_order=4, &
bins=1000, sg_halfwidth=1, output_file="%s.B1.csr", output_last_wake_only=1, &
slice_analysis_interval=5

B2: csrbend, angle=-0.0483456, l=0.5000484, e2=0, e1=-0.0483456, synch_rad=1, &
Nonlinear=1, n_kicks=51, integration_order=4, &
bins=1000, sg_halfwidth=1, output_file="%s.B2.csr", output_last_wake_only=1, slice_analysis_interval=5

B3: csrbend, angle=-0.0483456, l=0.5000484, e1=0, e2=-0.0483456, synch_rad=1, &

```

Nonlinear=1, n_kicks=51, integration_order=4, &
bins=1000,sg_halfwidth=1,output_file="%s.B3.csr",output_last_wake_only=1, &
slice_analysis_interval=5

B4: csrscsbend, angle=0.0483456, l=0.5000484,e2=0, e1=0.0483456, synch_rad=1, &
Nonlinear=1, n_kicks=51, integration_order=4, &
bins=1000,sg_halfwidth=1, output_file="%s.B4.csr",output_last_wake_only=1, &
slice_analysis_interval=5

W01: WATCH,FILENAME="watch.01.sdds"

W02: WATCH,FILENAME="watch.02.sdds"

W03: WATCH,FILENAME="watch.03.sdds"

W04: WATCH,FILENAME="watch.04.sdds"

!drifts:

!=====

Do : csrDRIF, L=5.004859, dz=0.1,use_stupakov=1

Di : csrDRIF, L=1.0, dz=0.1,use_stupakov=1

Df : csrDRIF, L=2.0, dz=0.1,use_stupakov=1

q: charge,total=1.0e-9

!beamlines

!=====

BC2 : LINE=(Q,B1,w01,Do,B2,w02,DI,B3,w03,Do,B4,w04,Df)