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EUROPEAN FEL Design Study

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Deliverable Title: Beam dynamics simulations through diagnostic elements: slice emittance measurement simulation

Task: DS-1

Authors: B. Marchetti, C.Vaccarezza

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The slice emittance of the electron beam in the transverse phase space is a crucial parameter in a FEL experiment. In this paper we describe the simulation of this measurement as it will be performed at SPARC in the first months of 2008.

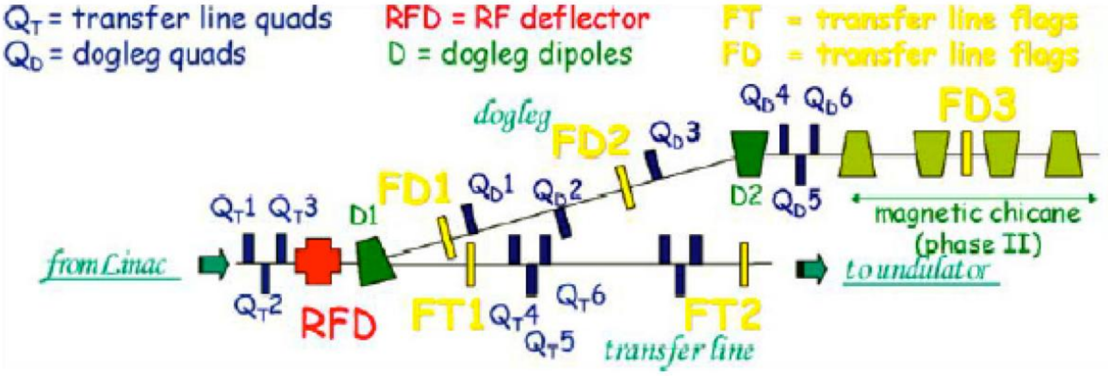

Figure 1: SPARC Transfer Lines

For the slice emittance measurement a quadrupole triplet, a RF-deflector and a flag are needed. Referring to the SPARC beamline layout, shown in Fig 1, we use for this scope the triplet Q1, Q2, Q3 to set the beam waist and the flag FT2.
The RF deflector allows measuring the beam transverse emittance along the bunch length; it gives to each slice of the bunch a kick in the vertical direction proportional to its longitudinal position with respect to the bunch centre, i.e. projecting the longitudinal beam distribution on the vertical plane. After the deflector, at the position of the flag, the transverse kick results in a transverse displacement of the centroid of each bunch slice.

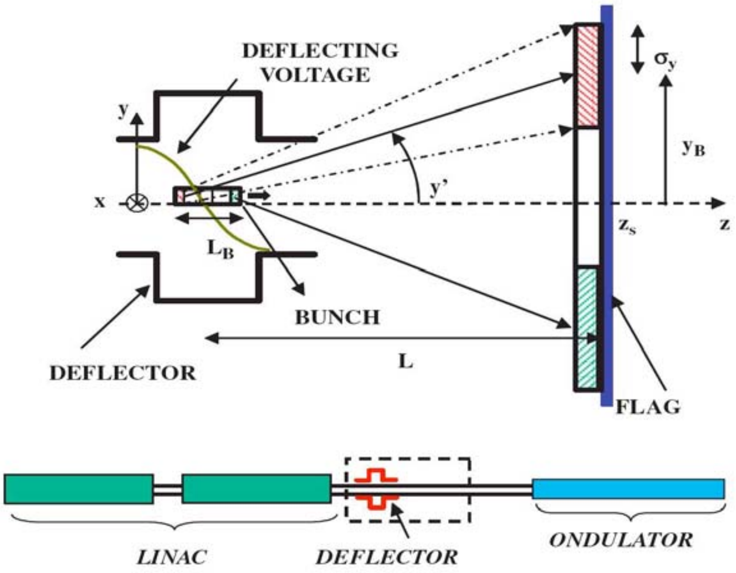

Figure 2: Schematic sketch of the RF-deflector working principle.

The image that appears on the flag when the deflector is “turned on” can be divided into slices. Performing a common quadrupole scan and analyzing the slices separately, keeping the variances of their horizontal projections, we can reconstruct the emittance of each of them.

This measurement has been simulated in order to test the program that will be used for the data analysis.

A beam represented by 1.5 x 10^5 particles obtained from PARMELA simulation at the end of the SPARC LINAC section has been tracked with the ELEGANT code along the SPARC transfer lines.

The phase space coordinates of the particles on the flag were the input for the programs we are going to describe.

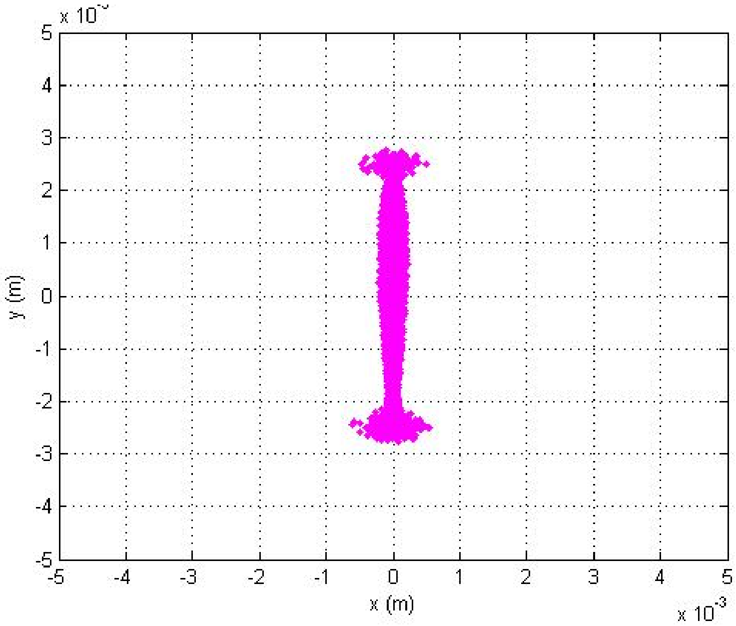

Figure 3: Pixel image of the electron beam as obtained from its particle coordinates via the Matlab code

A Matlab code was written to create an image in pixels which corresponds to a set of particles coordinates at the flag point, Fig. 3. The code produces a matrix output whose elements are proportional to the pixel intensity. The program needs the following input parameters:

- X and y coordinates of the particles at the flag point;
- Pixels dimension (they are assumed squared);
- Horizontal and vertical dimensions of the flag;
- Flag color level number.

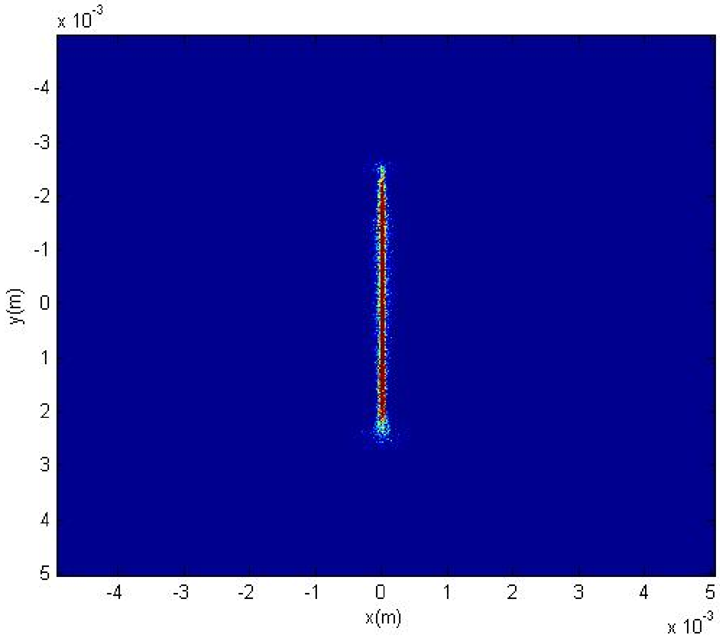

Figure 4: Electron beam pixel image with noise.

A second Matlab code has been written to simulate the noise on the flag, Fig. 4. It produces a new matrix with the following input parameters:

- The matrix correspondent to the clear image, produced by the previous program;
- The mean absolute value of the baseline noise;
- The window within which the random noise fluctuate.

A baseline of 10 particles per pixel is used and a window of 6 particles per pixel, which means that the noise in a pixel is between 7 and 13 particles.

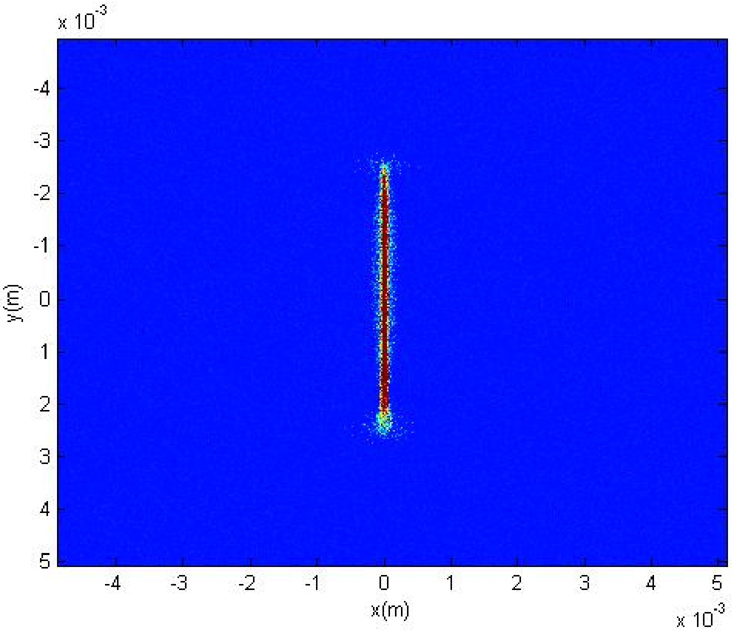

Figure 5: Range Of Interest Image (ROI) image selection

The first step in the image analysis is the selection of the Range of Interest (ROI): it is the area of the image which contains the beam signal, in order to cut off the noise of the remaining image portion Fig. 5. A different ROI for each slice in the bunch will be defined.

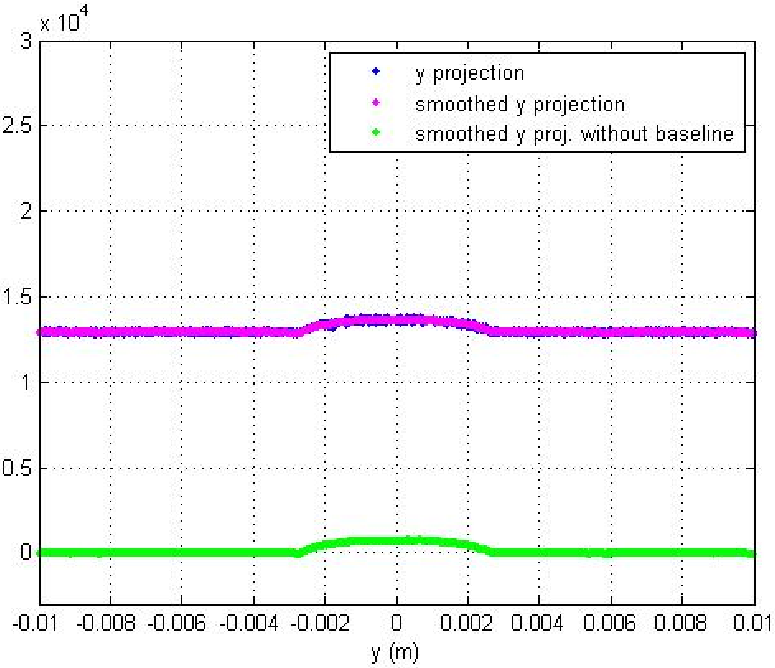

Figure 6: Vertical projection of the flag signal: row data (blue curve), smoothed data (magenta), smoothed data after the baseline subtraction (green)

At the beginning we analyze the vertical (y) projection of the signal, where y is the vertical axis of the flag image. We calculate the baseline averaging it in the extreme points and next we subtract this value from the initial projected signal. The new signal is smoothed and the maximum value is taken (Fig. 6). Than we consider a threshold value (which is a fraction of the maximum value, for example maximum/4) and only the signal over this threshold is in the ROI.

The ROI will be divided in N slices of equal length, where N is an arbitrary integer number that has to be chosen according to the RF-deflector resolution and the cooperation length value in the FEL experiment. From this moment each slice will be analyzed separately.

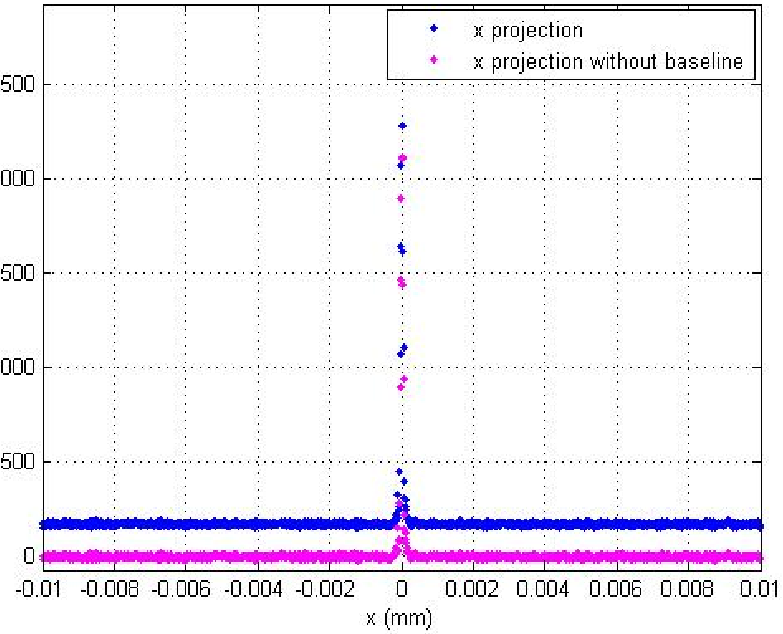

Figure 7: Horizontal projection of the flag signal: row data (blue curve), smoothed data after the baseline subtraction (magenta)

Now we want to select the x-ROI of a slice, where x is the horizontal axis of the flag image. This operation needs more attention because we want to recognize precisely the distribution tails in order to have a small error in the variance reconstruction.

After we have subtracted the baseline, the x-projection of the slice is fitted with the Gaussian function:

$$y = a1 * exp\left[-\left(\frac{x-b1}{c1}\right)^2\right].$$

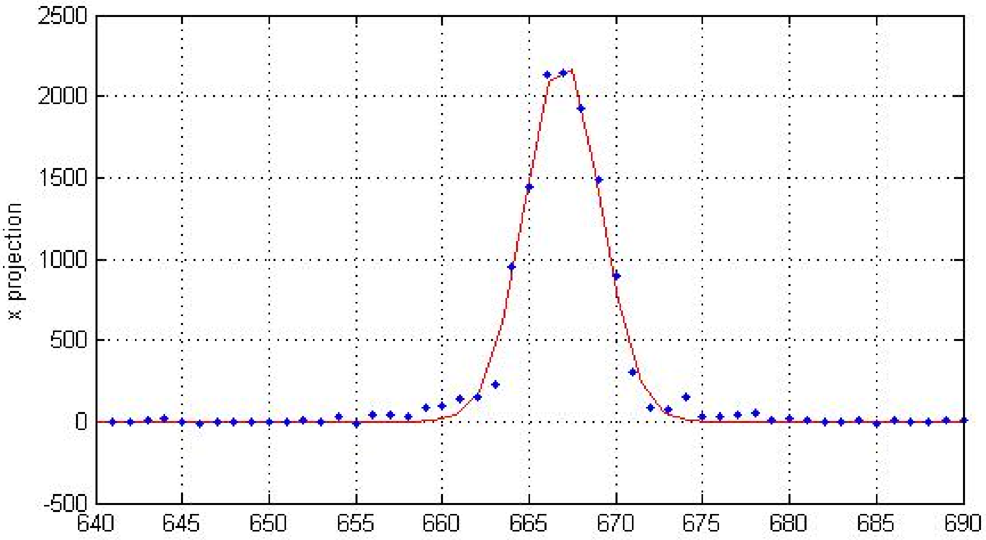

Figure 8: Gaussian fit of the horizontal signal

We know the centre of the distribution, b1, from the fit. Now the signal is smoothed and, starting from its centre, moving towards the tails, we look for the first two negative values (one on the left and one on the right). We recognize as ROI the portion of the non smoothed signal between these two points. This has been done because the negatives values of the smoothed distribution are exclusively due to the noise.

The cleaned signal is used to calculate the slice variance through formulas:

$$A = \sum_{i=1}^{n_j} a_i;$$

$$\overline{x} = \frac{1}{A}\sum_{i=1}^{n_j} a_i x_i;$$

$$< x^2 >= \frac{1}{A}\sum_{i=1}^{N} a_i(x_i - \overline{x})^2 = \frac{1}{N}\sum_{i=1}^{N} a_i x_i^2 - \overline{x}^2;$$

For the emittance reconstruction we considered SPARC line with the quadrupoles Q4, Q5 and Q6 switched off, the quadrupoles Q2 and Q3 switched on with a constant current correspondent to the focusing constants: K2= 3.067993 m^{-2} and K3= 7.103807 m^{-2} . In conclusion we had a varying quadrupole followed by a doublet of constant quadrupoles and a long final drift. The total matrix of the line was obtained by the products of the three matrices correspondent to these three parts of the line. All the quadrupoles were considered with their long lens matrix and the only variable in this calculus was the focusing function of the first quadrupole, k.

We used this matrix to obtain the relationship between the variances of each slice and k, which is:

$$< x^2 >= s22a^2 + 2s12ab + s11b^2$$

Con:

$$a = \left(6.00004Cos(0.1\sqrt{k}) + \frac{5.3699Sin(0.1\sqrt{k})}{\sqrt{k}}\right)$$
$$b = \left(5.3699Cos(0.1\sqrt{k}) - 6.00004\sqrt{k}Sin(0.1\sqrt{k})\right)$$

This function was used to fit the variance versus k distributions. From the fitting parameters we reconstruct the emittance through the:

$$\epsilon = \sqrt{\sigma_{1,11}\sigma_{1,22} - \sigma_{1,12}^2}$$

that contains the fit parameters.

In the following figures the results of the analysis are shown. Fig 9 shows the comparison between the normalized beam emittance (“input emittance”), as obtained from the beam tracking and the reconstructed emittance obtained with the quadrupole scan, using the variances of the particles on the flag.
In Fig. 10 the reconstructed emittance is reported as obtained analyzing only one noising beam image at the flag for each quadrupole configuration of the scan.
Fig. 11 shows the reconstructed emittance obtained averaging the results of the analysis of 32 noising images.

Conclusions
The beam slice emittance measurement has been simulated in order to test the program that will be used for the data analysis.
From the obtained results here described we can see that the error in the reconstruction is about 1% in the meddle of the bunch while in the tail it reaches 13%. This fact is caused by the different particles distribution in the slices: the central slices show a better ratio signal/noise because their signal x-projection are higher and with short tail respect to the external slice one.

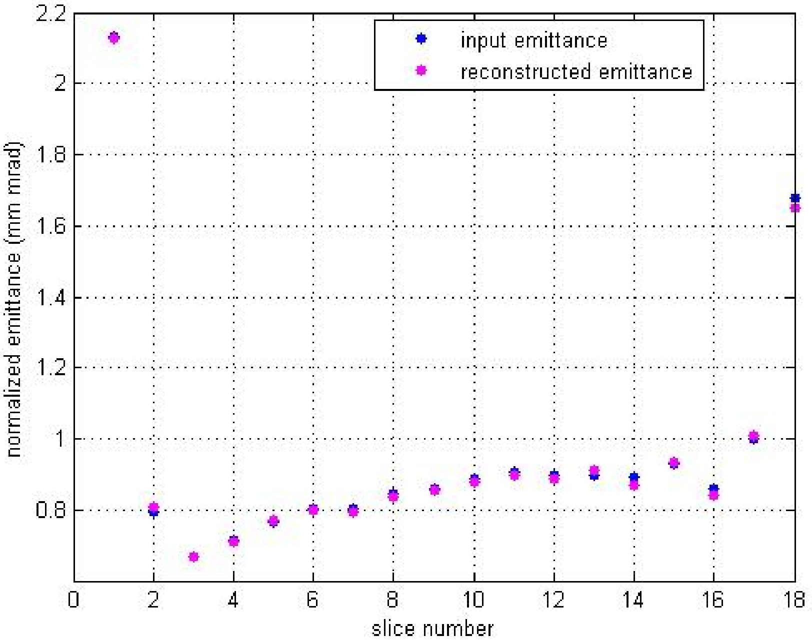

Figure 9: Comparison between the normalized beam emittance ("input emittance"), as obtained from the beam tracking, and the reconstructed emittance, obtained with the quadrupole scan , using the variances of the particles on the flag.

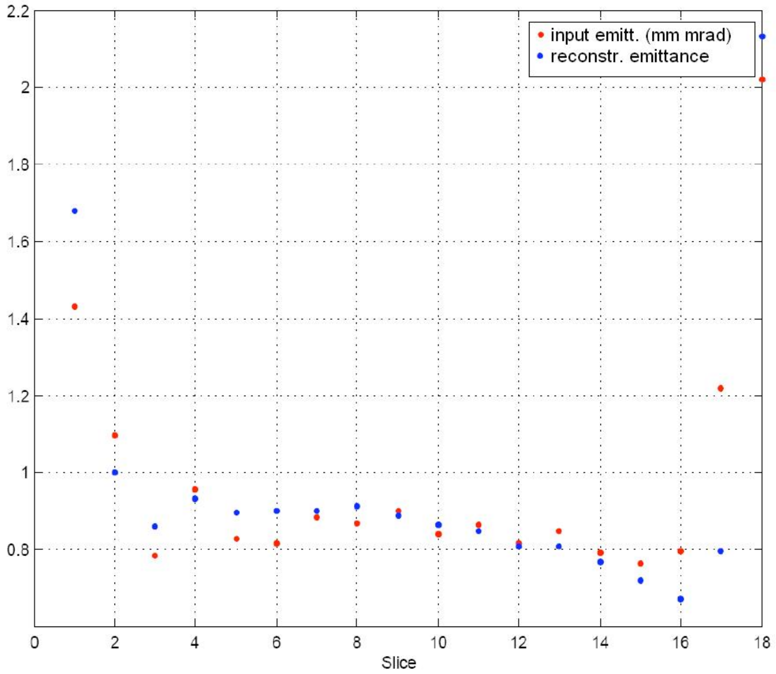

Figure 10: Reconstructed emittance obtained analyzing only one noising beam image at the flag for each quadrupole configuration of the scan.

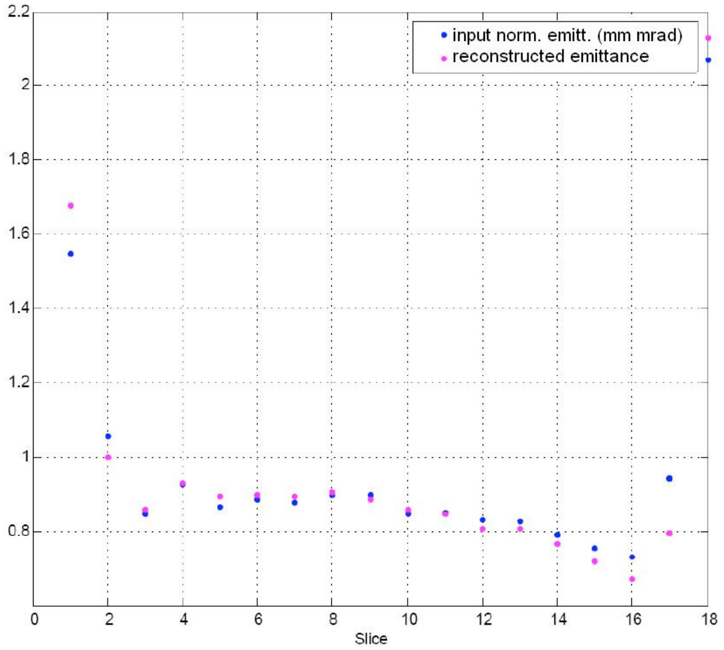

Figure 11: Reconstructed emittance obtained averaging the results of the analysis of 32 noising beam images.